A modified ray theory for predicting the $V(x,z)$ response of a point-focus acoustic microscope in the presence of a crack

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A modified geometric ray approach is used to predict the $V(x,z)$ or the acoustic material signature of a circular point-focus scanning acoustic microscope for an isotropic surface containing a surface-breaking crack. The microscope response is assumed to consist of a specularly reflected geometric contribution, and the leaky contribution due in part to surface waves scattered from the crack. The long thin straight crack is presumed to be characterized by symmetric reflection and transmission coefficients. This approach is numerically simple and can be used to evaluate the response of acoustic lenses of various geometries. The method is used to predict line scan $V(x,z_0)$ response of a microscope in the vicinity of the crack for several defocus distances z_0 and the results are compared with measurements.

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INTRODUCTION

The scanning acoustic microscope using a spherical lens is often used to image surfaces containing abrupt defects such as surface-breaking cracks. The presence of a thin crack perturbs the acoustic material signature mostly by scattering leaky Rayleigh waves that are excited on the sample surface by the probing acoustic beam. This scattering mechanism may be used to quantitatively characterize defects in nondestructive evaluation applications. It is therefore useful to create a simple theoretical model that can be used to predict the microscope response. Of particular interest are line-scan measurements of transducer voltage, called $V(x,z_0)$, obtained by keeping the lens at a constant defocus distance $z = z_0$ and scanning in x perpendicular to the defect. Crack displacement x is defined as the distance between the lens axis and the crack and $z = 0$ is defined at the geometric focus.

It is also important to model the $V(x,z)$ for the purpose of extracting the crack surface wave reflection coefficient. Somekh *et al.*[1] extended Atalar's[2] approach to cracked surfaces measured with a cylindrical lens. Ahn *et al.*[3] have used a Green's function solution implemented by a numerical boundary element method. None of these approaches involve the reflection coefficient of the crack in a simple way that permits a direct determination of its value.

In this work geometrical ray techniques are extended to include diffraction effects in the lens rod and are used to model the $V(x,z)$ for the circular aperture lens.

The total voltage response $V(x,z)$ of the microscope to a surface containing a crack may be expressed as

$$V(x,z) = V_G(z) + V_T(x,z) + V_{\text{refl}}(x,z), \qquad (1)$$

where $V_G(z)$ is the component of transducer voltage resulting from the specularly reflected rays, $V_T(x,z)$ is the component due to the leaky waves transmitted through or past the crack, and $V_{\text{refl}}(x,z)$ is the contribution due to the leaky waves reflected from the crack. The ray paths involved in determining these contributions are shown in Fig. 1. Here both the crack and the lens are assumed to have inversion symmetry, thus $V(x,z) = V(-x,z)$, i.e., the same response is detected on either side of the crack. It will be assumed that when a surface wave strikes a crack, a portion of the incident wave is transmitted with a field transmission coefficient T, and a portion is reflected with a field reflection coefficient R. These coefficients will be assumed here to be independent of the angle of incidence of the surface wave.

In this work, the crack is assumed to be long compared to the diameter of the beam spot insonifying the sample surface. For the lens and operating frequency considered here as well as a reasonable defocus distance, say $|z| < 500\ \mu$m, the length of the crack would have to be greater than 500 μm.

As the crack is assumed to be of infinitesimal thickness, the component $V_G(z)$ is unperturbed by the crack and hence is independent of x. Here $V_G(z)$ is evaluated using the ray model of Bertoni[4] modified to include the effects of diffraction in the lens rod.[5] The effective transducer field illuminating the lens aperture is solved numerically using the diffraction theory of piston radiators of Tjotta and Tjotta[6] and the result is approximated by a sum of Gaussians having the form $\Sigma_i A_i \exp(-\rho^2/B_i^2)$. Using a two-Gaussian approximation, the expression for $V_G(z)$ is found to be

$$V_G(z) = -\frac{T_{LW}(0)\,T_{WL}(0)\,\mathscr{R}(0)}{R_a^2 M}$$

$$\times \exp[2ik_w\,(D/n + f + z)]$$

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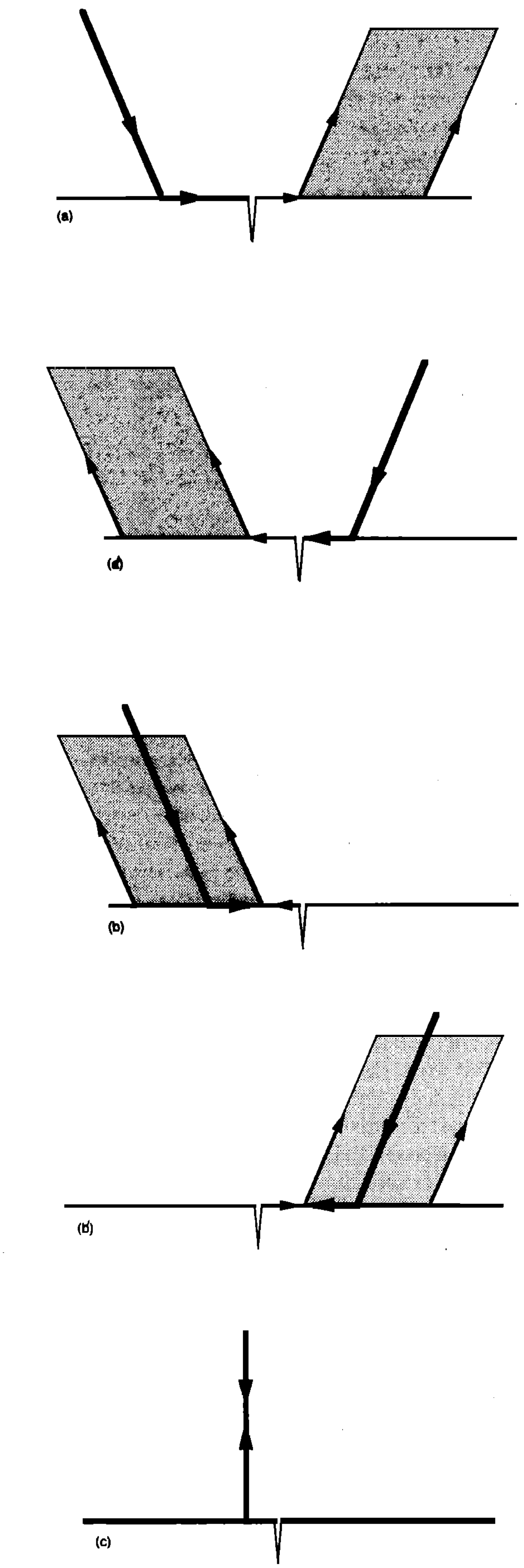

FIG. 1. Ray paths involved in $V(x,z)$ response for: (a) surface ray transmitted through the crack, (b) surface ray reflected from the crack, and (c) bulk wave specularly reflected from the surface.

$$\times 2 \sum_l A_{1l} \frac{n(B_{1l})^2(-u)}{ik_w(B_{1l})^2 - 2n(-u)} \times \left[\exp\left(\frac{ik_w r_T^2}{2n(-u)}\right) \exp\left(\frac{-r_T^2}{B_{1l}^2}\right) - 1 \right], \quad (2)$$

with

$$A_{1l} = A_i A_j,$$

$$\frac{1}{B_{1l}^2} = \frac{1}{B_l^2} + \frac{1}{(B_j M)^2},$$

$$i = 1,2, \; j = 1,2, \text{ and } l = 1,2,3,4,$$

where $T_{LW}(0)$ and $T_{WL}(0)$ are the lens–water and water–lens transmission coefficients at the angle $\theta = 0$, which corresponds to the normally incident rays, $\mathscr{R}(0)$ is the water–sample reflection coefficient at normal incidence, D is the transducer–lens aperture separation, R_a is the aperture radius, f is the focal length of the lens in water, and n is the acoustic index of refraction of the lens material relative to water. Additionally, $M = |f + 2z|/f$, $u = f(f + 2z)/2nz$, and

$$r_T = \begin{cases} R_a M, & z < 0 \\ R_a, & z > 0. \end{cases}$$

An alternate approach proposed by Auld[7] uses the electromechanical reciprocity relations to evaluate the transducer voltage response that is found to be the integral of the product of the scattered field and the incident field. The surface of integration may be chosen as any surface lying between the transducer and the fluid–sample interface. This procedure leads to an expression similar to that developed by Atalar.[2]

In this work both incident and scattered fields are evaluated using ray theory. This asymptotic approach loses its validity in the neighborhood of caustic points. A study of the ray paths between the sample and the transducer as described in the following reveals the presence of several caustic points whose location varies with defocus distance z. One must therefore be careful to choose a surface of integration to avoid these regions. For this reason the geometrically scattered component $V_G(z)$ is evaluated in the back aperture plane which leads to expression (2), while the leaky wave contribution $V_T(x,z)$ and $V_{\text{refl}}(x,z)$ are evaluated in the plane of the transducer as described below.

I. SURFACE WAVE SCATTERING FROM A CRACK

A. Transmitted component

Leaky wave excitation and re-radiation are not reciprocal in the following sense. A uniform plane wave incident on a solid substrate at the Rayleigh critical angle launches a leaky Rayleigh wave. On the other hand, a leaky Rayleigh wave that is propagating along the fluid–solid interface radiates a nonuniform leaky beam into the fluid rather than a plane wave. Therefore, the excitation of and radiation from leaky waves are treated separately.

Figure 2 illustrates the leaky ray structure for a defect-free sample when the lens is closer than the focal distance or

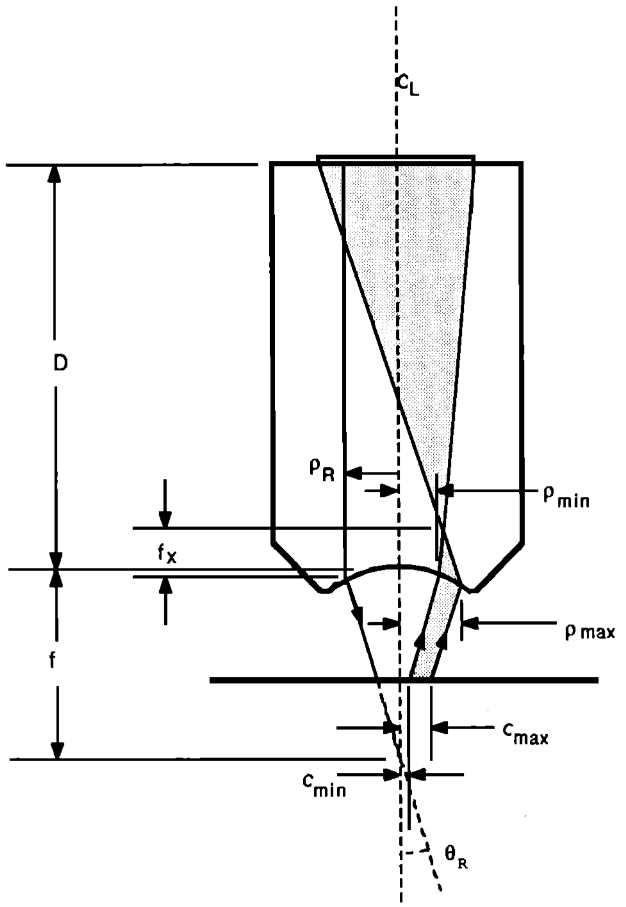

FIG. 2. Leaky ray bundle in the x–z plane. The shaded region here represents the part of the leaky ray flux that excites the transducer. The total excitation flux is obtained by rotation about the lens axis.

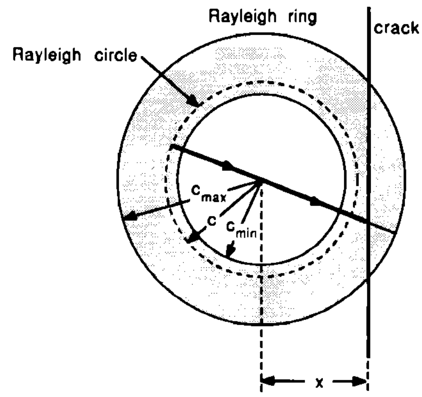

FIG. 3. A surface ray launched on the left that excites the transducer by leaking energy within the Rayleigh ring resulting in the leaky flux as shown in Fig. 2. As the ray crosses the crack the leaky field is reduced by the factor T.

$z<0$. Rays that cross the lens surface at a distance from the lens axis $\rho = \rho_R$ strike the sample surface at the Rayleigh critical angle θ_R and excite leaky Rayleigh waves propagating in the plane of incidence. The axial symmetry of the lens dictates that all such launch points on the sample surface form a circle, called the Rayleigh circle, of radius $c = |z|\tan\theta_R$. The surface waves that are so excited focus at the point of intersection with the lens axis and then diverge beyond. Within the lens rod the leaky rays form a ring-shaped focus of radius ρ_R, located a distance f_x above the lens surface. The leaky-ray bundle reaching the transducer is formed by the flux of rays that originate from a ring-shaped region on the sample surface bounded by circles of radii c_{min} and c_{max}, as illustrated in the figure by shading. For brevity this region will be referred to as the Rayleigh ring. The resulting component of $V(z)$ is independent of x and is called $V_L(z)$.

When a crack is present and is sufficiently displaced from the lens axis, some surface rays are leaked into the couplant without having been scattered by the crack, and therefore contribute to the transducer voltage as in the defect-free case. However, the component of voltage due to those rays that are transmitted through the crack is scaled by the crack transmission coefficient T, as shown in Fig. 3. The transmitted contribution $V_T(x,z)$ is given by

$$V_T(x,z) = V_L(z)\cdot(Q_R(x,z) + Q_L(x,z)), \tag{3}$$

where $V_L(z)$ is the contribution to $V(z)$ due to the leaky surface waves for the defect-free surface, and $Q_R(x,z)$ and $Q_L(x,z)$ are fractional contributions to $V(x,z)$ from the transmitted rays re-radiated into the couplant on the right and left sides of the lens, respectively, as shown in Fig. 1. In the absence of a crack $Q_R = Q_L = 0.5$. Bertoni[4] had found the leaky contribution to be

$$\begin{aligned} V_L(z) = & -T_{LW}T_{WL}\exp[i2k_w(D/n+f)] \\ & \times\left(\frac{2\alpha_L\lambda_W(\sqrt{\pi}f^3\sin\theta_R)^{1/2}}{(n\lambda_W D)^{3/4}}\right)e^{i\pi}K \\ & \times\exp(i2k_w(D/n+f+|z|\cos\theta_R) \\ & \times\exp(-2\alpha_L|z|\tan\theta_R), \end{aligned} \tag{4}$$

where

$$\begin{aligned} K = & \frac{2e^{(-i\pi/4)}}{N_T{}^2}\int_0^{N_T}[\exp i(N-N_R)^2 \\ & + \exp(-i\pi/2)\exp i(N+N_R)^2]\sqrt{N}\,dN. \end{aligned} \tag{5}$$

In (5), $N_T = \sqrt{k_w/2nD}\,R_T$ and $N_R = \sqrt{k_w/2nD}\,\rho_R$, where R_T is the transducer radius, α_L is the real part of the Rayleigh pole of the water–sample reflection coefficient, and expressing ρ_R in terms of the lens radius of curvature R_0,

$$\begin{aligned} \rho_R = & (f-R_0)\sin\theta_R\cos\theta_R \\ & + \sin\theta_R\sqrt{(f-R_0)^2\cos^2\theta_R - (f^2-2R_0f)}. \end{aligned}$$

Transmission coefficients T_{WL} and T_{LW} are evaluated at $\rho = \rho_R$. Diffraction effects suffered by the transducer field in the lens rod may be included by scaling $V_L(z)$ by the value of the transducer field $U(\rho)$ at the lens aperture evaluated at $\rho = \rho_R$.

The $V_T(x,z)$ contribution to the $V(x,z)$ will be assumed to be proportional to the total field flux leaked into water from the area bounded by c_{min} and c_{max} in Fig. 3. This assumption is most reasonable for a relatively long lens rod where the phase variation of the field across the transducer is small. The factor Q_R can be seen to be

$$Q_R = \tfrac{1}{2}(Q_1 T + 1 - Q_1), \tag{6}$$

where Q_1 is the fraction of the ray flux intercepted by the crack and therefore scaled by the crack transmission coefficient T, and $(1 - Q_1)$ is the fraction of the ray flux that is unscattered. Under these conditions, the factor Q_1 can be expressed as

$$Q_1 = \frac{\text{area of obstructed part of Rayleigh ring}}{\text{total area of Rayleigh ring}}. \tag{7}$$

The above relation views all leaky radiation source points as contributing equally to $V_T(x,z)$. Since the total area of the Rayleigh ring is $\pi(c_{\max}^2 - c_{\min}^2)/2$ it follows from (7) that

$$Q_1 = \begin{cases} 1, & 0 < x < c_{\min}, \\ \dfrac{4}{\pi(c_{\max}^2 - c_{\min}^2)} \displaystyle\int_{c_{\min}}^{c_{\max}} \rho \cos^{-1}\left(\frac{x}{\rho}\right) d\rho, & c_{\min} < x < c_{\max}, \\ 0, & x > c_{\max}. \end{cases} \tag{8}$$

By applying a similar analysis to the surface rays that are excited on the right and collected by the transducer on the left side of the crack we have

$$Q_L = \begin{cases} 0.5\,\dfrac{2\phi T + 2(\pi/2 - \phi)}{\pi} = \dfrac{\phi T + \pi/2 - \phi}{\pi}, & 0 < x < c, \\ 0.5, & x > c, \end{cases} \tag{9}$$

where $\phi = \cos^{-1}(x/c)$.

The transmitted component $V_T(x,z)$ can be viewed as the leaky component $V_L(z)$ that is attenuated in part by the crack. The above permits determination of $V_T(x,z)$ for $z < 0$. When $z > 0$, the surface waves diverge away from the lens axis and, thus, no transmitted leaky radiation excites the transducer, consequently, $V_T(x,z) = 0$ for $z > 0$.

B. Reflected component

The reflected contribution to $V(x,z)$ has two components, defined by

$$V_{\text{refl}}(x,z) = V_{RR}(x,z) + V_{RL}(x,z), \tag{10}$$

where V_{RR} and V_{RL} are the contributions to $V(x,z)$ due to the surface waves reflected by the crack incident from the right and left, respectively. For $x > c$ and $z < 0$ the crack is outside the Rayleigh circle and the Rayleigh waves are incident on the discontinuity only from the left, thus $V_{RR} = 0$. Define $V_{\text{refl}}(x,z) = V_R(x,z)$ in this range. First, $V_R(x,z)$ will be found and then both $V_{RR}(x,z)$ and $V_{RL}(x,z)$ will be expressed in terms of $V_R(x,z)$ for all x and z.

In order to determine the various field amplitude factors of the Rayleigh rays due to focusing and phase delays, we assume a ray field of amplitude $|U(\rho_R)|$ is incident on the lens surface at the critical radius $\rho = \rho_R$ where it is refracted into the couplant to strike the sample surface at $\rho = c$. The value $|U(\rho_R)|$ is calculated using the Bessel function expansion of a piston radiator field used by Tjotta and Tjotta[6] as opposed to assuming a uniform illumination of the lens aperture as it provides better agreement for $V(z)$ with measurements in the near focal region.[5] Spherical focusing by the lens and propagation in the lens rod and water introduces the amplitude factor ρ/c and the phase factor $\exp ik_w(D/n + f + z\sec\theta_R)$, see Fig. 2. Propagation along the surface to the crack results in the phase factor $\exp ik_p(x - z\tan\theta_R)$, where k_p is the pole of the water–substrate reflection coefficient, $k_p = k_R + i\alpha_L$ (Ref. 2). Reflection from the crack modifies the field by the factor R. Traversing the return path to the transducer causes a duplication of the phase changes involved in forward propagation as well as an amplitude factor $\exp - \alpha_L(x - z\tan\theta_R)$ due to leakage into water. Transmission through the lens–water interface is taken into account by using the field transmission coefficients $T_{LW}(\rho_R)$ and $T_{WL}(\rho_R)$, which can be found, for example, by the method of equivalent networks of Oliner *et al.*[8] In addition, as shown by Bertoni,[4] there is a coefficient E associated with the excitation and re-radiation of the surface waves, given approximately by

$$E = -2\alpha_L \sqrt{\frac{2\pi|z|}{k_w \cos^3\theta_R}} \exp\left(-\frac{i\pi}{4}\right). \tag{11}$$

The focusing and spreading of surface rays for several Rayleigh wave ray paths is depicted in Fig. 4. The surface waves pass through a focal point where the lens axis intersects the sample surface and then strike the crack, located a distance x from the lens axis, which acts as a linear mirror with a field reflection coefficient R. The surface waves reflected from the crack appear to propagate from a virtual focus a distance $2x$ from the lens axis denoted as F in Fig. 4. These reflected waves propagate to the left until they reach the neighborhood of the Rayleigh circle, where the ray fields that leak into the couplant are collected by the lens and excite the transducer. Ray spreading in the plane of the sample introduces a field amplitude factor $\sqrt{c/(c+2x)}$ and, in addition, the planar focus at the lens axis introduces a phase change of $-\pi/2$ (Ref. 9).

The surface wave rays (e.g., FB, FA, and FC in Fig. 4) radiate into the couplant, each ray thereby creating a parallel family of rays launched at the angle θ_R. Since the parallel rays in the plane do not spread, there is only angular spreading in the horizontal planes. When the radiated ray bundle reaches the lens, it will have a radius of curvature of $\rho_R + 2x$, and thus a change of amplitude of $\sqrt{(c+2x)/(\rho_R+2x)}$ results from ray spreading in the couplant.

Since the bundle of radiated rays spreads only in the angular direction, passage through the spherical lens results in two astigmatic foci of focal lengths f_x and f_y, given by

$$f_x = f - R_0 \text{ and } f_y = (f - R_0)\rho_R/2x + f_x. \tag{12}$$

For the lens used in this work and for $x > c$, it follows that $0 < f_y < D$ and, thus, there is a focus in the lens rod. The ray field therefore suffers a phase change of $-\pi/2$ due to propagation through each of the two foci as well as additional factors given approximately by $\sqrt{f_x/D}$ and $\sqrt{f_y/|D - f_y|}$ due to ray spreading in the lens rod. The above approximations assume $f_x, f_y \ll D$ and $f_x \approx f/n$, reasonable for the lens used here.

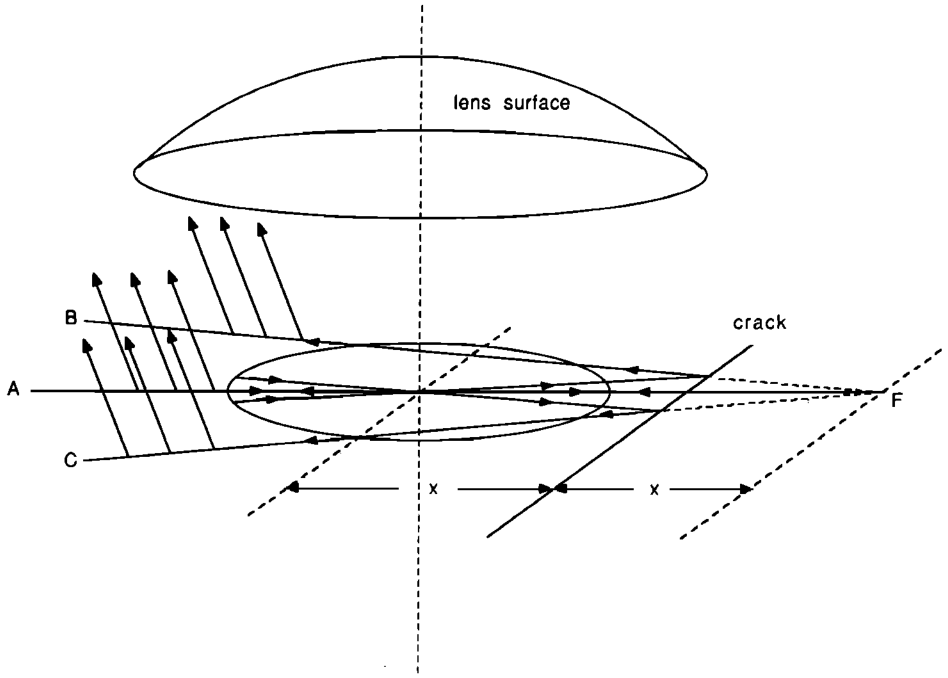

FIG. 4. Focusing and divergence of surface waves reflected from the crack. The circle on the surface is the Rayleigh circle of radius $c = |z|\tan\theta_R$.

Finally, the astigmatic beam illumination of the transducer has a phase curvature of

$$\exp\frac{ik_w(x-\rho_R)^2}{2nD}\cdot\exp\frac{ik_w y^2}{2n(D-f_y)}.$$

Combining the above factors and integrating the resulting field over the transducer, the voltage response V_R is found to be

$$\begin{aligned}V_R(x) = {} & |U(\rho_R)|RT_{LW}(\rho_R)T_{WL}(\rho_R)E\\ & \times\exp 2ik_w(D/n+f+z\sec\theta_R)\\ & \times\exp 2ik_p(x-z\tan\theta_R)\\ & \times\exp\left(-\frac{i3\pi}{2}\right)\sqrt{\frac{ff_x}{n}}\frac{\rho_R}{D\sqrt{2|x|c}}K_1(x),\end{aligned} \quad (13)$$

where

$$\begin{aligned}K_1(x) = {} & \frac{1}{\pi R_T^2}\int_{\text{transducer}}\sqrt{\frac{D}{D-f_y}}\exp\frac{ik_w(x_1-\rho_R)^2}{2nD}\\ & \times\exp\frac{ik_w y_1^2}{2n(D-f_y)}dx_1\,dy_1.\end{aligned} \quad (14)$$

The total reflected voltage contribution $V_{\text{refl}}(x,z)$ will now be considered when the crack lies within the Rayleigh circle, i.e., $x<c$ and $z<0$. The rays incident from the left give the same expression as found for $V_R(x)$ except that for $f_y > D$ there is only one astigmatic focus in the lens rod, which corresponds to $0<x<2(D-f_x)/(f-R_0)\rho_R$. In this case the ray field does not suffer a $-\pi/2$ phase shift due to passing through the focus in the corresponding range of x, or explicitly,

$$V_{RL}(x) = \begin{cases} V_R(x)\exp(i\pi/2), & 0<x<2\rho_R(D-f_x)/(f-R),\\ V_R(x), & \text{otherwise.}\end{cases} \quad (15)$$

To find the contribution due to rays incident on the right, $-x$ should replace x in the expression (10). It should also be noted that since $f_y<0$ for $|x|$ small and $x<0$, there is only one real focus in the lens rod and therefore

$$V_{RR}(x,z) = V_R(-x,z)\exp(i\pi/2). \quad (16)$$

It can be seen that

$$V_{RR}(x,z)=0, \quad \text{for } z>0,\ x>0$$

and

$$V_{RL}(x,z)=0, \quad \text{for } z>0,\ |x|<c. \quad (17)$$

For $z>0$ and $x>c$, the diverging beam incident on the sample surface has passed through the spherical lens focus and has therefore suffered a phase shift of $-\pi$. However, there is no real Rayleigh wave focus on the sample surface in this case since the incident beam is diverging as it strikes the sample surface and hence the phase shift of $-\pi/2$ included in the expression for $V_R(x,z)$ no longer applies. Thus, for $z>0$

$$V_{\text{refl}}(x,z) = \begin{cases} V_L(x,z) = V_R(x,z)\exp(-i\pi/2), & x>c,\\ 0, & x<c.\end{cases} \quad (18)$$

II. THE TOTAL VOLTAGE RESPONSE—COMPARISON WITH MEASURED DATA

Measurements of $V(x,z)$ were made using a 60-mm-long × 30-mm-diam fused silica lens rod excited by a 5-mm-diam $LiNbO_3$ p-wave transducer operating at 50 MHz. The focusing end has a spherical cavity of 3-mm radius of curvature and a 5-mm aperture. Water was used as a coupling fluid between the lens and the sample surface, which in this case was a glass microscope slide containing a surface-breaking crack.

Values of $V(x,z)$ were calculated by evaluating expression (1) at several defocus distances z_0. Approximate values of ρ_R, $c_{\min}$, and $c_{\max}$ were found using a simplified geometric model. The values of the reflection coefficient R and the transmission coefficient T were assumed as $R = 1$, $T = 0$ independent of the angle of incidence, a reasonable assumption for small x.

Measured and predicted $V(x,z)$ are illustrated in Fig. 5 for two values of the defocus distance z, viz., 300 and 400 μm over a range $0 < x < 500$ μm. It should be noted that the calculated $V(x,z)$ are scaled so as to match the measured values at large x.

The theoretical model described above predicts that for x near the origin, or equivalently, the lens approximately centered above the crack, f_y is large as given by (12), and there is only one line (cylindrical) focus in the lens rod located a distance f_x above the lens aperture. Actually, the crack acts as a plane mirror and for a symmetrical lens the reradiated ray structure in the fluid becomes indistinguishable from the ray structure in the uncracked sample. This results in a ring and not a line focus in the lens rod as was described above. Thus disagreement between theory and experiment is expected for x small as can be observed in Fig. 5(a).

The crack reflection coefficient R and the transmission coefficient T were assumed to be independent of the angle of incidence. For large x the transducer is excited by only those surface rays that are nearly normally incident on the crack. For smaller values of x, the transducer is excited by the more obliquely scattered rays. Thus, the effective crack scattering coefficients should be expected to depend on x, with R decreasing and T increasing with increasing x. Note that the ripple due to reflected surface waves in Fig. 5 which is proportional to R, actually decreases in amplitude faster than predicted by the ray theory in which R is assumed constant.

III. CONCLUSIONS

A ray model was developed to predict the response of a point focus scanning acoustic microscope for a surface containing a linear surface-breaking crack. It is assumed that the specularly reflected ray fields are unperturbed by the thin crack while the surface wave scattering by the crack is characterized by field reflection and transmission coefficients that are reciprocal and independent of the angle of incidence. The spreading and convergence of bulk and surface rays are treated by the geometric ray theory.

The expressions derived for the various voltage components are relatively simple and have the advantage of being simpler to implement numerically than those derived by alternate methods. The resulting predictions are shown to compare favorably with measurements for a spherical lens with a circular aperture.

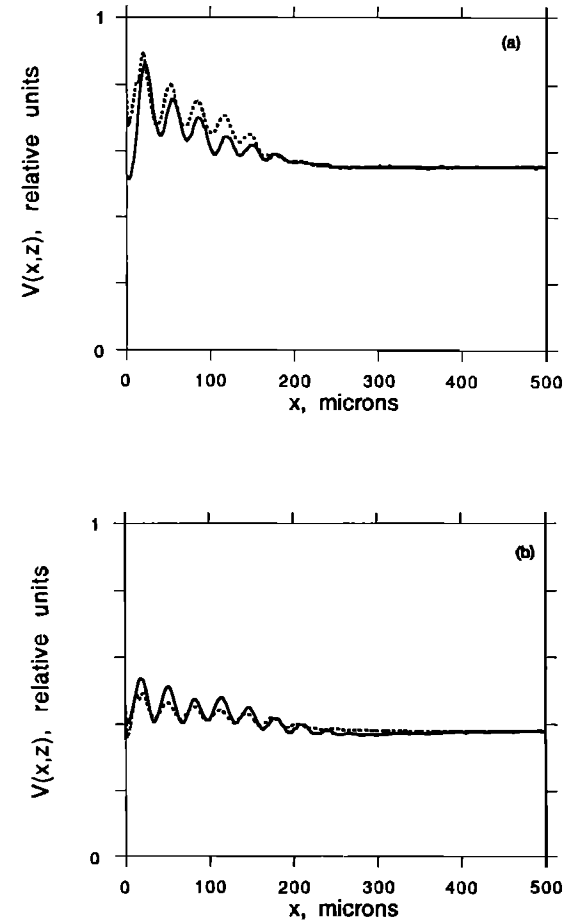

FIG. 5. $V(x,z = z_0)$ for a lens having a circular aperture. The measured values are denoted by the solid line while the theoretical prediction is dashed: (a) is for $z_0 = -300$ μm and (b) is for $z_0 = -400$ μm.

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